Use of GPS in unification of vertical datums and detection of levelling network errors

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ABSTRACT

We discuss the interconnection of vertical networks using GPS and gravimetric geoid models, and give some examples how this can be done in practice. We also discuss several error sources of GPS-based methods which degrade the accuracy of height determination. GPS can be used for interpolation of heights, and some medium/low precision spirit levelling may be replaced by GPS. It can also be utilised in maintaining country-wide levelling networks or connecting national levellings. However, many ordinary spirit levelling errors cannot be controlled by GPS and it will not replace precise levelling over relatively short distances.

INTRODUCTION

During recent years, use of GPS in height determination has increased rapidly. This brings forward the question whether slow and expensive levelling can be replaced by GPS, or, at least, the levelling errors can be controlled with it. There are two different things which we have to take into account: the accuracy of the GPS itself and the accuracy of the geoid model we need to transform heights above the ellipsoid to orthometric or normal heights. In the following we discuss some aspects of the GPS in height determination.

The observing geometry and the way the errors affect GPS measurements are most unfavourable to the vertical component. The unmodelled (and unknown) troposphere errors are one of the most important accuracy limiting factors today. Even in permanent GPS networks, the residual errors can amount to up to 0.01 ppm scale error (*M. Ollikainen*, private communication, 1999) and the height component can vary in the cm-range. We will discuss this later in this paper.

The land uplift, caused by the post-glacial rebound, is a typical phenomenon in the Fennoscandian area. The maximum uplift value, about 10 mm/yr., is at the end of the Gulf of Bothnia. We must correct the height values for the uplift from the standard epoch of the height datum to the epoch of the GPS observations. For this, we can use e.g. the uplift values given in the map of Kakkuri (1991).

The time-independent part of the luni-solar tide should be treated in a consistent way in spirit levelling, geoid model and GPS computations if orthometric (or normal) heights are to be obtained from GPS observations. We may distinguish three different cases of tide corrections: *Non-tidal geoid* and *non-tidal crust* when the tide is fully removed, *mean geoid* and *mean* *crust* when the tide is retained, and *zero geoid* when the attraction of the Sun and the Moon is removed but the permanent tidal deformation is retained. For the crust, there is no analogy to the zero geoid; the ”zero crust” would be the same as the mean crust. For an extensive discussion on the permanent tide in GPS observations, see Poutanen *et al.* (1996).

The current GPS programs give coordinates reduced to the non-tidal crust. The levelling, however, refers to various geoids, depending on the country. In Table I we reproduce the summary table of Ekman (1995). The differences between various geoids/crusts amount to up to 10 cm in the area of Fennoscandia but we can easily convert all quantities e.g. to their mean values using formulae of Ekman (1989):

$$\Delta H_m - \Delta H_n = 0.296\ g(\sin^2 j_N - \sin^2 j_S) \quad [\mathrm{m}] \qquad (1a)$$

$$N_m - N_n = (1 + k)(0.099 - 0.296 \sin^2 j) \quad [\mathrm{m}] \qquad (1b)$$

$$\Delta h_m - \Delta h_n = -0.296\ \boldsymbol{h}\ (\sin^2 j_N - \sin^2 j_S) \quad [\mathrm{m}]. \qquad (1c)$$

The first formula is used to convert height differences of the non-tidal crust above the non-tidal geoid to height differences of the mean crust above the mean geoid and is appropriate for treating levelling. The second formula converts the non-tidal geoid heights above the ellipsoid to the mean geoid heights and the third formula converts height differences of the non-tidal crust above the ellipsoid to the height differences of the mean crust, and is used for GPS ellipsoidal heights. Above, g k and $\boldsymbol{h}$ are the Love numbers, and j_N and j_S refers to the latitude of northernmost and southernmost station, respectively.

In each case, the *Love numbers used in the original non-tidal calculations must be applied*, regardless how close to or far from reality they are. E.g. in the Swedish levelling g= 0.8 has been used, for the OSU89B spherical harmonic model k = 0.3, and $\boldsymbol{h}$ = 0.609 in the Bernese GPS software; already these three are incompatible among themselves.

To obtain orthometric or normal height from GPS observations one needs a geoid model. There are several good models available in the Fennoscandian area, like the Nordic Standard Geoid NKG-96 (R. Forsberg, private communication, 1997), BSL95A and FIN95 (Vermeer, 1995). Additionally, there are also several global geoid models, like OSU91A and EGM96 but these are less accurate locally. The NKG-96 is a cm-geoid without any major distortions due to a considerably better gravity coverage of the surrounding area than its predecessor NKG-89. Its level was adjusted using results of several GPS campaigns.

Tidal correction of these geoid models is sometimes a bit complicated question because it depends on the underlying global potential spherical harmonic expansion. However, most of these seem to be of a non-tidal type.

Country Height system	geoid	geoid type	epoch
Finland N60, orthometric	classical	mean	1960
Sweden RH70, normal	quasi	non- tidal	1970
Norway NN 1954, orthometric	classical	mean	none
Denmark DNN GI, orthometric	classical	non- tidal	1950
W.Europe UELN 73, orthometric	classical	mixture	(1960)
E.Europe Kronstadt, normal	quasi	?	?

Table I. *European height systems (Ekman 1995).*

All the above-mentioned geoids are quasi-geoids and thus heights referring to these are normal heights. At seashore both the orthometric height H_{ort} and the normal height H_{norm} are equal because geoid and quasi-geoid coincide. However, the difference increases with terrain height by (Heiskanen and Moritz, 1967)

$$H_{norm} - H_{ort} = \frac{C}{\bar{g}} - \frac{C}{\bar{g}} \approx \frac{\Delta g_B}{98\,2000[\mathrm{mgal}]} H_{ort} \tag{2}$$

where C is the geopotential number, $\bar{g}$ is the mean gravity, $\bar{g}$ is the mean normal gravity, and Dg_B is the Bouguer anomaly. One can compute exact difference if geopotential numbers are available, otherwise the difference can be estimated with the Bouguer anomaly.

Global geoid models are not fitted to a particular height system. Therefore, one needs a local adjustment (height and tilt) of a geoid to use it in height determination with GPS. Geoid models may have long periodic (several hundreds of km) errors which are not removed in such simple adjustment. There are some pre-fitted models, like FIN95 which is adjusted to the Finnish N60 system. However, also these are time-dependent. E.g. FIN95 is approximately in epoch 1993 because it is the mean epoch of the GPS observations used in adjustment. In precise work, one has to either reduce the observed ellipsoidal heights to the epoch of the geoid model by using the known uplift values or make a new local adjustment of the geoid.

UNIFICATION OF VERTICAL DATUMS

Small local levelling networks, like those of cities or municipalities can be connected to national levelling networks in a straightforward way. This is not true anymore in connecting national networks. It is an oversimplification to compare height systems of two countries just by giving one number for the height difference. We have to more or less arbitrarily choose what values are intercompared. As an example, when speaking about Swedish and Finnish heights, we first convert Finnish orthometric heights to normal heights which are consistent with the gravimetric (quasi-)geoids available. At the seashore both heights are equal and we need not to distinguish there between them.

Next we convert Swedish heights to refer to the mean geoid instead of the non-tidal geoid and finally, the land uplift correction brings the heights to the same epoch (from year 1960 in Finland, 1970 in Sweden). After this procedure there is not so much left of the original systems and one may ask, with a good reason, if there is any sense to make this.

A more reasonable way to do the connection is to establish a new, well defined height system for the area and compute transformation parameters from the old national systems to the new one. In this we follow the guidelines shown in Ekman and Mäkinen (1991 and 1995; hereafter E&M) where they introduce the *Nordic Height System 1960* (NH60). However, for practical reasons, especially from the viewpoint of GPS, we refine their proposal slightly. For brevity, we call it here NEH2000 (*The North European Height System 2000*):

1. The zero point is the *NAP* (Normaal Amsterdams Peil).
2. The heights are *normal heights*.
3. The permanent tide (time average of the tidal deformation) is retained so the crust refers to the *mean crust*. The geoid is the *mean geoid*.
4. The normal heights are reduced from the national system epoch to the epoch 2000.0 using the *land uplift relative to the geoid* (i.e. the sum of the apparent uplift and the eustatic rise of the sea level). The absolute uplift which can be obtained with GPS, can be converted to this ”levelled” uplift by subtracting the rise of the geoid.

In the following we will shortly discuss the advantages and disadvantages of these choices.

1. The selection of the NAP as the zero level agrees with the definition in E&M and is consistent with the UELN 73 zero point. In the European-wide EUVN campaign (Ihde *et al.*, 1998) it is also a natural choice and may also give a good connection to the proposed World Vertical Datum (Rapp, 1995). There are some theoretical arguments to use the potential W_0 to define the zero level (e.g. Grafarend and Ardalan, 1997) but in practice there is always a strong tendency to retain the old definitions.
2. In their paper, E&M used geopotential numbers. Their aim was mainly to study the sea surface topography and with the small heights there is no need to distinguish between the normal and orthometric height. While geopotential numbers are required for the NEH2000, the question of their conversion to metric units remains. The gravimetric geoids we use nowadays are often quasi geoids, i.e. basically height anomaly maps, and one should use normal heights with them. The difference between the normal height and the orthometric height is less than 10 cm in most places in the Fennoscandian and Baltic states area because the heights are so small. Especially, with GPS, when the user wants "heights above the sea level", one uses an available geoid model and then obtains normal heights directly. The common intuition about "the height above the sea level" is more relevant with the orthometric height, and the normal height concept conflicts this intuition. We admit that physically orthometric height is the natural height system and there is no principled reason not to use it *if* a good geoid becomes available. There are strong opinions in favour and against of both systems and the topic requires a lot more discussion, especially among the groups who compute global or regional geoids. In this paper we look at the topic in the viewpoint of current practice and data availability.
3. There is an extensive discussion on GPS and tide in Poutanen *et al.* (1996) where the use of mean geoid and mean crust is proposed. In some cases the zero geoid could be theoretically better but the mean geoid has the advantage that it describes the temporal mean of the actual, instantaneous equipotential surface corresponding to the mean sea level. The definition thus implies that GPS observations should be reduced from the non-tidal crust to the mean crust. One should abandon the physically irrelevant non-tidal concept in geodetic measurements. One should also note that the reference surface of the levelling is defined in a different way in different countries as shown in Table I, and the heights H should be converted to refer to the mean geoid.
4. The heights in the Fennoscandian and the Baltic area are changing due to the postglacial rebound of the crust. In order to maintain the correct connection to the global networks, the absolute land uplift should be applied because the height above the ellipsoid, h, is changed by the absolute uplift. However, with normal heights one has to use the levelled uplift value, i.e. the geoid rise is to be subtracted. The magnitude of the geoid rise, however, is relatively small and can be neglected in all but the most accurate nation-wide measurements. If one uses GPS to obtain the absolute uplift value, the geoid rise can be been taken e.g. from Ekman (1993). The magnitude of the rise is 5 – 10 % of the uplift value. The epoch 2000 is quite close to the ending of precise levellings in the Nordic countries. It is now proper time to discuss updating the national height systems.

REALISATION OF A COMMON VERTICAL DATUM

A more dense network is needed to firmly connect a vertical datum, like NEH2000 to the national height systems like N60 or RH70. We show only some guidelines here, mostly based on the discussion above and references therein. The selection of the geoid has the key role here. We have a good situation in Fennoscandian area because even two excellent geoids are available, NKG96 and BSL95A. Both are fixed to a common datum, using GPS data as described above. In the following, we choose the NKG96 for the basis of the NEH2000 realisation because it is slightly better in the Southern Baltic than the BSL95A (Martin Vermeer, private communication, 1997).

The GPS data set we use here contains a total of 95 points in Finland, Sweden and Estonia. The network can be seen in Fig. 1 where we show the post-fit residuals. According to the guidelines above, we made the following corrections to the data values:

H: The heights were reduced from the national datum epoch (FIN 1960, S 1970) to epoch 2000 for land uplift using the uplift values from Kakkuri (1991) and for the eustatic rise of the sea level, value 1mm/yr.

h: The values obtained from GPS were first corrected from non-tidal crust to mean crust, using (1c). After that the uplift of about 5 years (from the mean epoch of all measurements to 2000) was added. Additionally, geoidal height *N* was converted from non-tidal to mean height. After this, we computed

$$r = h_{GPS}^{2000} - N_{mean} - H_{lev}^{2000} \qquad (3)$$

and fitted a polynomial surface

$$S = \sum_{i=0}^{m} \sum_{j=0}^{n} a_{ij} (l - l_0)^i (j - j_0)^j \qquad (4)$$

separately for each country. The post-fit residuals are shown in Fig. 1, and the coefficients of the fit in Table II. In the adjustments we used the value l_0 = 60° and j_0 = 20° for all three countries.

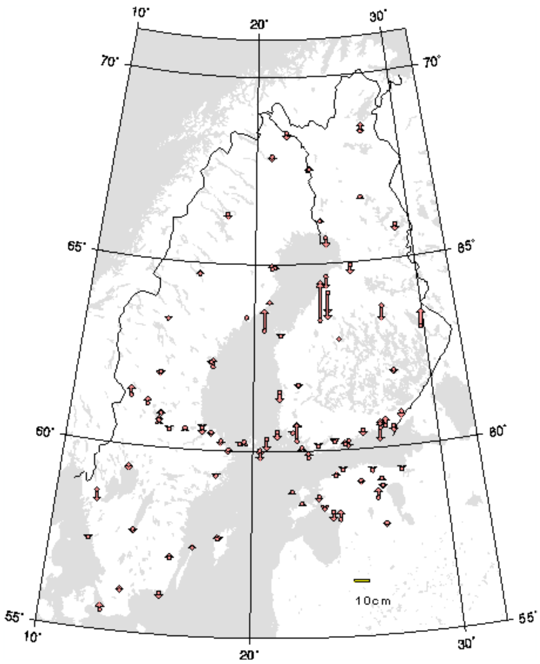


Figure 1 *Post-fit residuals. A second degree polynomial surface was fitted separately to the height difference H_{GPS} – H_{lev} in each country and the arrows shows the residuals of this fit. Except some outliers, all residuals are well*

Finland

S =.17012968E+00 ± .34890958E-01
+ X*.47965032E-01 ± .18969820E-01
+ Y*.37165633E-01 ± .17007044E-01
+ X^2*-.77663047E-02 ± .22111091E-02
+ X*Y* .56553564E-02 ± .23894568E-02
+ Y^2*-.53395461E-02 ± .20179607E-02

Sweden

S =.13171358E+00 ± .22111548E-01
+ X*.39942825E-01 ± .79216262E-02
+ Y* -.47656854E-01 ± .13020401E-01
+ X^2* -.60229776E-02 ± .94603853E-03
+ X*Y*.75005216E-02 ± .19034318E-02
+ Y^2* -.48163915E-02 ± .16280249E-02

Estonia

S =.39838716E-01 ± .19115648E+00
+ X*-.37574914E+00 ± .17291180E+00
+ Y* -.41951423E-02 ± .50219724E-01
+ X^2*.61359686E-01 ± .66604315E-01
+ X*Y* .32666889E-01 ± .18979704E-01
+ Y^2* .37674007E-02 ± .41947902E-02

Table II *Coefficients of the fit of Eq. (4). Here we have marked X = j – j_0, Y = l – l_0. j_0 = 60°, l_0 = 20° in all cases.*

below 10 cm.

We did not use the geoid rise because of its smallness; also the land uplift in Estonia was neglected because the datum epoch is not well defined and because the country is relatively small in area. Also no correction was applied to convert Swedish normal heights to refer to the mean geoid. Finnish heights were not converted to normal heights. These corrections are either small or will be absorbed into the fit. The idea was to minimise the steps for practical purposes, still not unnecessarily lose accuracy.

In conversion of national heights to a unified datum (NEH2000, EUVN, ...), one should first make the uplift correction to get the heights in the epoch of the datum. After this, using (4) and coefficients from Table II, a datum shift can be done. These steps can be done automatically if one uses a land uplift model, either a grid from which a value can be interpolated, or a (high order) polynomial surface for less accurate applications.

ERRORS AND ERROR CONTROL

Formal errors of a GPS solution do not agree with the actual errors of the measurement but are normally far too optimistic. In the following we discuss some of the errors in GPS height determination and how they affect the accuracy of height determination with GPS.

Troposphere related errors are more difficult to eliminate than the ionosphere errors because troposphere affects in the same way both L1 and L2 frequencies. Measurement of the water vapour content via the signal path is not possible in routine measurement. Use of a standard troposphere model may result in an error which affects both scale and height. The scale error *Dl / l* is

$$\frac{\Delta l}{l} = \frac{\Delta r}{R_{\oplus} \cos z_{\max}} \qquad (5)$$

where *Dr* is the troposphere model error, $R_{\oplus}$ is the Earth's radius and z_{max} is the maximum observed zenith angle. The height error *Dh* amounts to

$$Dh = dDr_{ab} / \cos z_{max} \qquad (6)$$

where dDr_{ab} is the relative troposphere error between points *a* and *b*. In Fig. 2. we show a scale error obtained in the Finnish permanent GPS network FinnRef (Ollikainen and Koivula, private communication 1999) which most likely comes from the troposphere model error. According to Eq. (5), only a ±2 cm error in troposphere refraction estimation is sufficient to produce the detected scale variation of about ±0.01 ppm. We have here no methods to see directly the total height error of Eq. (6), but there are good reasons to assume that it exists.

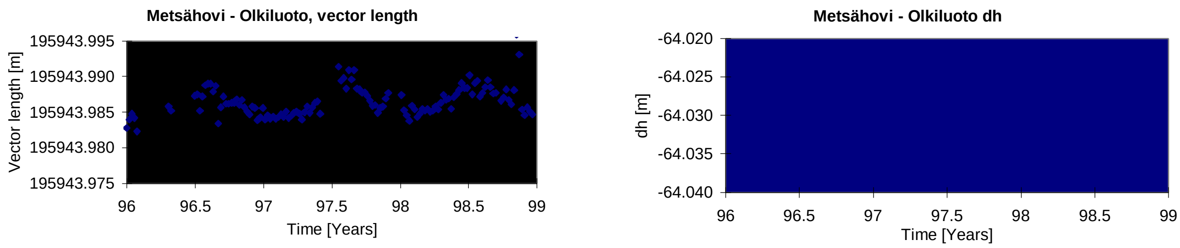


Fig. 2. *Change of vector length and height difference between the Finnish permanent GPS network stations Metsähovi – Olkiluoto. The change in vector length is visible in other vectors, too, and one can estimate the scale error to be of the order of 0.01 ppm. The change in height difference is somewhat bigger but we do not know if there is any constant bias. Both effects are most likely coming from minor errors in troposphere delay*

model and they have a clear yearly cycle. (M. Ollikainen and H. Koivula, 1999, private communication)

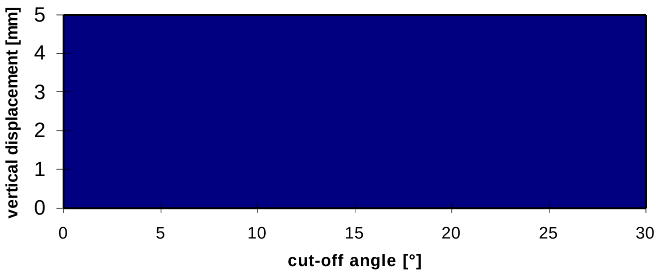


Fig. 3. *Effect of cut-off angle in computed height. The vector length was 36 m. Both antennae were Dorne Margolin type choke ring antennae but the other antenna had a radome, the other was without. The zero of the vertical scale is arbitrary. (Kylkilahti, 1999)*

Troposphere error grows rapidly at small elevation angles. Therefore, in most applications, the cut-off limit in GPS processing is set to 15°. There is also another elevation dependent error source, namely the antenna phase centre variation. The position of the antenna electrical phase centre depends e.g. on the direction and frequency of the incoming signal. If identical antennae are used in the whole network, the phase centre error cancels out almost totally but if there are different antennae (or even different antenna mountings, radomes, snow layer on top of an antenna, multipath...), the error remains. The phase centre variation causes a systematic error in height which cannot necessarily be seen in normal processing, although it can be even centimetres. It can be eliminated with a field calibration but this implies a lot of extra work. Antenna patterns have been determined e.g. for the FinnRef stations (Kylkilahti, 1999).

Also the change of cut-off angle affects the height component; we demonstrate the effect in Fig. 3. The reason for this is that the relative phase centre variation of two antennae is a function of elevation angle. In this example we get relatively small change because antennae were identical and only radome and mounting were different. If possible, cut-off angle should be kept in data processing unaltered from campaign to campaign.

Use of precise ephemeris is also crucial in height determination. In Fig. 4. we show the error in height when broadcast ephemeris are used instead (Ollikainen 1997). The behaviour shown in Fig. 4. is applicable to this particular campaign only. Change in SA pattern may cause different distance dependency in other campaigns.

This far we have been speaking on errors only. After this we may ask, what kind of accuracy can be achieved in GPS levelling. In Fig. 5. we give an example of this. The figure is taken from thorough study of Ollikainen (1997) where he tested GPS levelling on two areas in South Finland. He used different geoid models with and without local adjustment. When compared to the spirit levelled heights, the mean accuracy of the height differences with the best geoid models was ±15 mm. This includes also errors of the spirit levelling. When spirit levelling errors were removed, the mean accuracy of the GPS levelling was ±12 mm.

This kind of accuracy can be achieved even in a moderately wide area. On the other hand, accuracy may not improve remarkably in smaller networks because errors coming e.g. from antenna phase centre variation or multipath do not depend on distance.

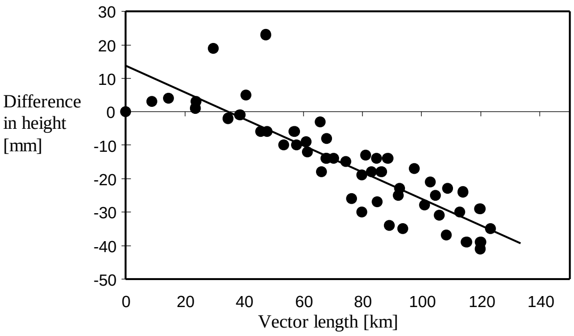


Fig. 4 *Example of height error caused when broadcast ephemeris are used instead of precise ephemeris. (Ollikainen 1997)*

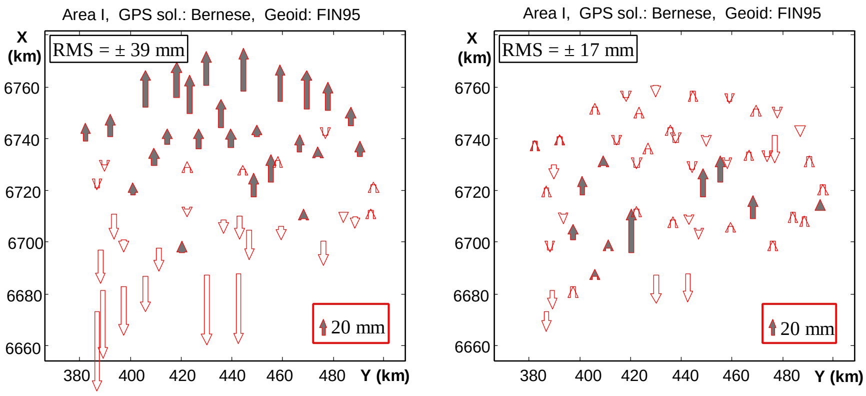
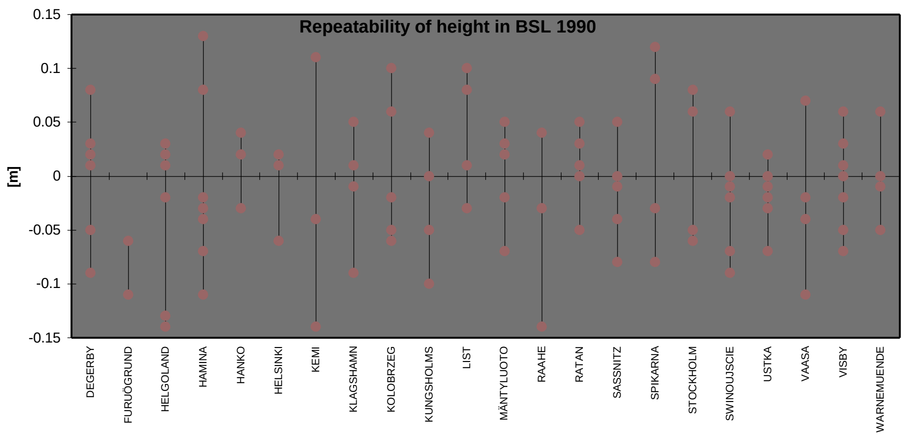
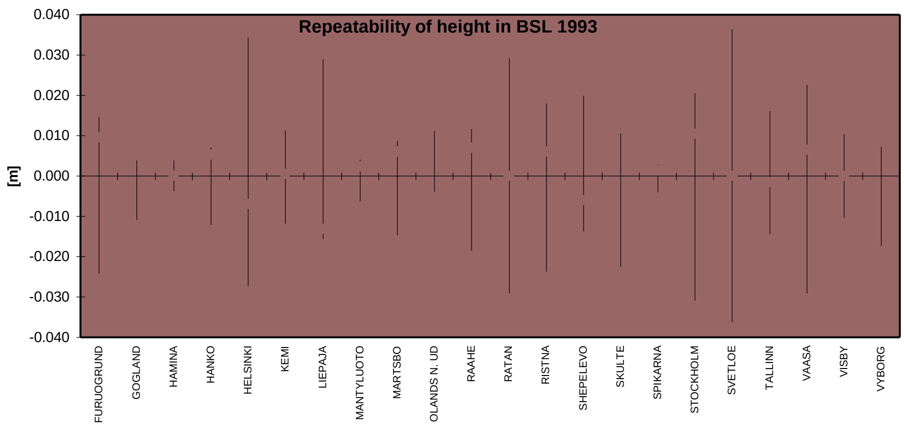


Fig. 5. *Height difference between GPS levelling and spirit levelling (left): using FIN95 geoid model; (right): after an additional plane fit (level and tilt). (Ollikainen 1997)*





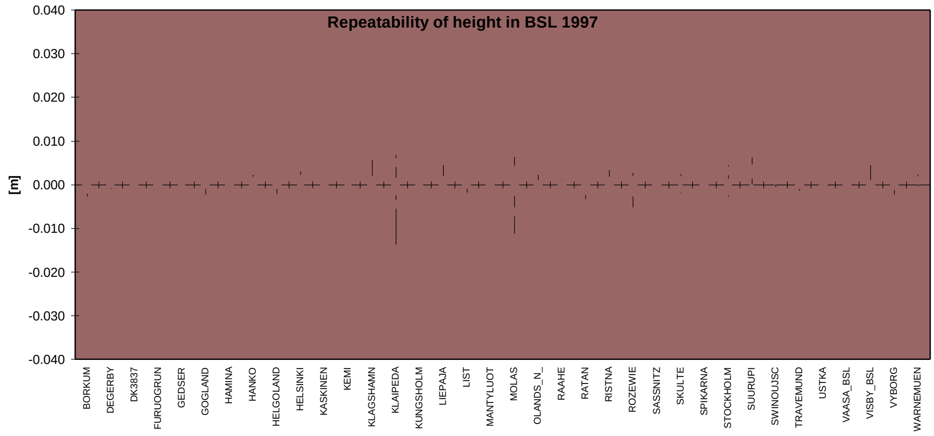


Fig. 6. *Repeatability of BSL 1990, 1993 and 1997 GPS campaigns in height component. Note the different scale in 1990 campaign plot. The repeatability of the 1997 campaign is about one tenth of that of the 1990 campaign, namely 1 mm in horizontal and 2.6 mm in the vertical component.*

CONCLUSION

During the nineties, there has been a vast development on receiver technology, precision of satellite ephemeris and data processing. As one might expect, the increase in accuracy has been as dramatic as the change in technology. We demonstrate this in Fig. 6. where repeatability in height of three successive Baltic Sea Level GPS Campaign is shown. However, the systematic errors can be larger than might be expected from the repeatability. In (Poutanen, 1995) we show an example from the 1993 campaign, where the solutions of different computing groups deviate more from each other than the repeatability of the results of one group, indicating unmodelled systematic errors.

In the current state-of-the-art the GPS determination gives a sub-centimetre accuracy in the height above the ellipsoid. The geoid has the key role in determining orthometric or normal heights with GPS. The error of the geoid could be even an order of magnitude bigger than the error in GPS measurement. Combination of other methods, tide gauge observations and satellite altimetry, gives us a way to improve the accuracy, and a method to fix the position and level of a geoid. Without a local adjustment, long wavelength errors may still remain.

The need for a common vertical datum is quite obvious in the future when GPS observations are extended over the border of a country. There are several ongoing projects aiming to this goal, like EUVN (European Vertical GPS Reference Network, Ihde *et al.*, 1998). We know that the proposed new systems could remain in scientific use only and their utilisation in common use is a long process requiring European-wide agreements and acceptance. The final decision of the new system will be political and economical, not a scientific one. If the level and geoid model of EUVN will be selected according to the general guidelines shown above, the NEH2000 heights will be directly in the datum of EUVN.

What could one say about the future of GPS in levelling? If we consider the whole error budget, including observational and computational errors, it is obvious that over relatively short distances precise levelling cannot be replaced by GPS. It is possible to replace low-precision levelling where accuracy requirements are in the cm range (see e.g. Ollikainen 1997, and Fig. 5.).

The same holds true in detecting levelling errors. If we speak about loop closure errors of mostly a few cm, it is doubtful if GPS can bring any new information, or the amount of work required could be almost as big as that of a partial relevelling. Many levelling benchmarks are in such places that a direct GPS measurement is not possible but an auxiliary marker has to be established. Connection of the benchmark and the temporary marker requires spirit levelling. If there are tens of new points, the amount of the additional work will be considerable.

GPS could be used in detecting gross errors in levelling network. On the other hand, one should be able to eliminate gross errors already during the levelling in all but very special cases (like water crossings or spike measurements) where GPS can be used as an extra check.

The situation changes when we speak about regional or country wide levelling networks. Levelling over distances of several hundred kilometres may take years (or decades) and even with an excellent formal error of 0.5–0.8 mm $km^{-½}$ the uncertainty at the ”other end” will be comparable to that which can be achieved with GPS in a few days. All this depends on how good geoid models will be available. On this basis, also unification of vertical datums can be done with the aid of GPS. GPS is also suitable for point densification inside a pre-existing network and invaluable in roadless areas where spirit levelling is impossible.

I can see the future of levelling as a mixture of traditional levelling and GPS. Local precise work is still done with traditional methods but the less accurate part will be replaced by GPS. National levelling networks could be maintained with a sparse permanent GPS network. The permanent stations are used as control points for local updates or densification either by traditional methods or by GPS. When the large Nordic precise levelling works end in 3–4 years from now, one should seriously discuss the role of permanent GPS networks, how the levelling networks can be maintained and how much of traditional levelling can be abandoned in the future. But certainly we will still need the levelling instruments.

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