SENSOR NOISE CHARACTERISTICS AND ERROR PROPAGATION: AN EDUCATIONAL APPROACH BASED ON COLLOCATED SMARTPHONE MEMS ACCELEROMETERS

Stathis C. Stiros[1], Georgia N. Fotopoulou, Christodoulos I. Glaros[3]

[1] Dept. of Civil Engineering, Patras University, Patras 26500, Greece (stiros@upatras.gr)
[2] Dept. of Civil Engineering, Patras University, Patras 26500, Greece (civ7460@upnet.gr)
[3] Dept. of Civil Engineering, Patras University, Patras 26500, Greece (civ7269@upnet.gr)

Key words: *accelerometer; MEMS; smartphone; drift; Law of Error Propagation; stochastic; noise; experiment*

ABSTRACT

We made a simple experiment to show the implications of the (usually ignored) Law of Error Propagation in accelerometers, which represent the most common instrument for structural and geotechnical engineering and for inertial positioning. Double numerical integration of the acceleration is used to compute displacements, which are characterized by drift, a gradually increasing error, and some deterministic techniques to filter this stochastic error are adopted. We used a simple, controlled experiment and 3-D MEMS accelerometers, included in all modern smartphones, to show some characteristics of the (stochastic) errors in computer displacements ("drifts"). Two fully collocated smartphones were attached in the wall or the floor of the cabin of a lift of multi-story buildings, and were forced to a linear controlled path, so that the errors in estimates of the displacements in three axes can be controlled. Displacement errors (drifts) are stochastic and can be described by the law of error propagation, while this experiment, in different variations, can be used even in classroom teaching.

Introduction

With the advent of modern electronic instruments and of modern computational techniques permitting collection and analysis of masses of data, various aspects of the theory of measurements and errors have been ignored, and measurements and measurement-derived data are usually assumed to be characterized by white noise only (cf. Moschas and Stiros, 2019; for a rare example see El Diasty et al 2008), while rather deterministic techniques are used to filter errors.

A method to highlight the non-random character of errors and of their propagation in modern instruments is to study drift in displacements derived from accelerometers, which represent the most common instrument for structural and geotechnical engineering (Boore and Bomer, 2005) and for inertial positioning (El Diasty et al 2008). Since acceleration is the second derivative of distance, double numerical integration of accelerometer data is widely used to compute displacements, for example during earthquakes. However, such displacements are characterized by a cumulative error in the form of a monotonous increasing function, known as "drift", occasionally of impressive amplitude (Fig 1), and different simple or more complicated techniques to remove this drift have been proposed (e.g. Boore and Bommer, 2005).

In this article we examine this problem from the point of view of the theory of errors, using simple controlled experiments that can be easily reproduced by anybody and anytime, without cost, using usually available

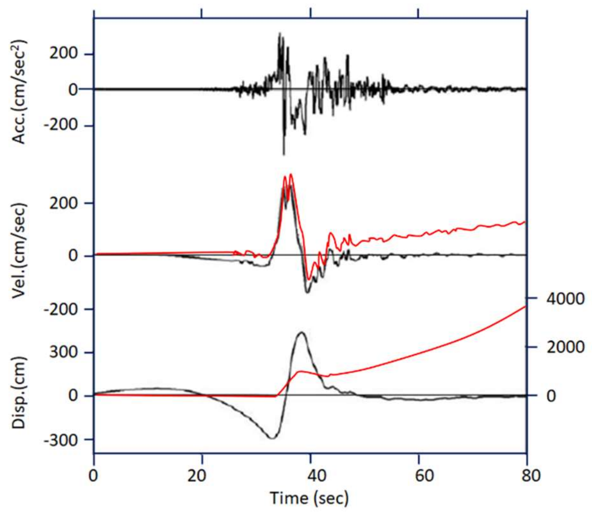

Figure 1. Acceleration, velocity and displacement for a station during the Chi-Chi 1999 Taiwan earthquake. Red curves indicate uncorrected velocity and displacement, known as "drift", a nearly monotonous increase of the error in computed velocity and displacement; the latter may reach unrealistic values, tens of meters after 80 seconds. Black curves indicate corrected time histories of velocity and displacement based on deterministic approaches, usually constrained by the known value of the drift at the last point of the time series, assuming no total displacement. This approach is satisfactory for earthquakes (short duration, usually no permanent displacement), but not for continuously moving objects (drones etc.). Modified after Boore and Bommer (2005).

hardware. These experiments can be modified to be made even in a classroom in different levels.
For our experiments we used MEMS accelerometers which are currently incorporated in miniature form in all modern smartphones. These can compute acceleration in an easily recognized cartesian system, while data collection and analysis with output in CSV files is easily made through freely available APPs. For the experiments, we used two different smartphones, side-by-side (fully collocated), fixed on the wall of the lift of an elevator. This permits to constrain well the displacement derived from the recorded acceleration. The overall approach is different from that used by various investigators to test the response of MEMS accelerometers (Evans et al, 2014; Moschas and Stiros, 2019)
This approach, based on simple, controlled experiments which can easily modified and adapted to local conditions, and on data analysis with simple, popular software (for example EXCEL) may shed light to the stochastic character of displacements derived from accelerometer data is an example of techniques that can revolutionize teaching in broad fields and at various levels, but can also be used in what is known as citizen response and contribution in science (cf. Lawrence et al 2014).

Methodology

Acceleration is the second derivative of displacement, and double acceleration of displacement leads to displacement. In the case of experimental data, we assume that during a certain time interval, an accelerometer has recorded v measurements of acceleration (γ_i, t_i) along a certain axis with a constant sampling interval τ at times t_i. We also assume that observations of acceleration γ_i are characterized by white noise of small amplitude (standard error σ_γ), so that observations of acceleration γ_i differ little from from the 'true" values (Fig. 1). We also assume zero acceleration and velocity at t=0.
As is explicitly analyzed in Stiros (2008), using numerical integration, velocity at t_v is described as the area between x-axis and the graph of accelerations between t=0 and t_v and is given by equation

$$\upsilon_\nu = \sum_{\iota=1}^{\nu} \tau\gamma_\iota \quad (1)$$

Similarly, displacement between t=0 and t_v is given by equation

$$u_\nu = \tau^2 \sum_{i=1}^{\nu} (\nu + 1 - i)\gamma_i \quad (2)$$

According to the law of error propagation, the typical error σ_f of a variable $f=f(\chi_1, \chi_2, \ldots\chi_v)$, which is a function of v uncorrelated variables $\chi_1, \chi_2, \ldots, \chi_v$ with random noise with typical errors $\sigma_{\chi 1}, \sigma_{\chi 2}, \ldots \sigma_{\chi v}$, is given by the equation

$$\sigma_f^2 = \sum_{i=1}^{\nu} \left(\frac{\partial f}{\partial xi}\right)^2 \sigma_{x_i}^2 \quad (3)$$

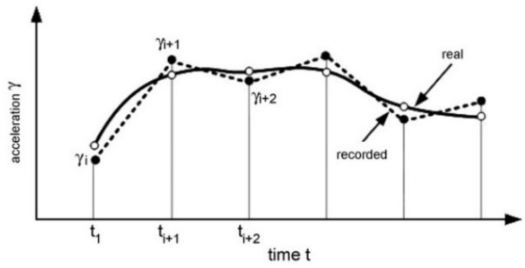

Figure 2. The "true" acceleration (smooth, continuous line in bold) is recorded/sampled as γ_i by the accelerometer at moment t_i, assumed with a constant rate. The area of each trapezoid formed by consecutive observations is summed till a certain moment t_v to compute the velocity υ_v at this specific moment. This corresponds to numerical integration. A similar process permits to compute displacements u_v from cumulative summation of velocities through double numerical integration. After Stiros (2008).

and after computations and simplifications, for large numbers of v, the uncertainty of displacement is given by equation

$$\sigma_{u_\nu} = 0.6\tau^2\nu^2\sigma_\gamma \quad (4)$$

This indicates that the error in the displacement u_v derived from an accelerograph increases with the square of the number of measurements, and that at a certain probability level it is located within a space defined by $(-\sigma_{u_\nu}, \sigma_{u_\nu})$.

Experiment

We used common cheap micro-MEMS 3-D accelerographs found in most smartphones, equipped with a common, cost-free App (for example Physics Suite© etc.) to record acceleration, usually in non-constant intervals and with a mean rate of 10-20Hz. The two smartphones were mounted side by side on the wall or the floor of an elevator cabin in a multi-story building, with their side parallel to a cabin corner. This signifies that the two smartphones were forced to record the motion of the elevator cabin, in intervals of movement and of standstill along the vertical axis.

For this reason, we had several constraints to the motion recorded by the two smartphones: the "true" overall path between floors, in various combinations, was easily recorded, intervals of movement and of no movement were alternating, while along the two horizontal axes the "true" displacement was null. This permits to compare displacement derived from accelerometers with their 'true" values.
We made several experiments using the collocated smartphones and recorded the acceleration of the elevator cabin while it was moving from one floor to the other in different combinations, noticing the exact track of the elevator. The experiment was repeated using different smartphones to avoid case-dependent results.

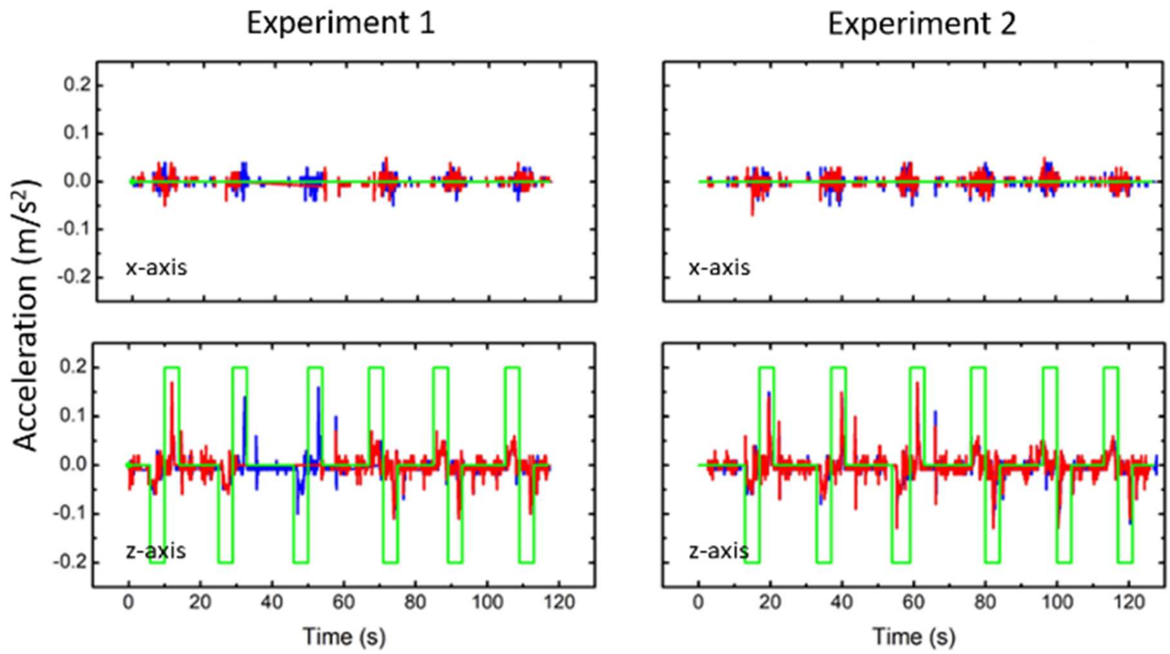

Figure 3. Recorded accelerations along a horizontal axis (x) and the vertical axis (z), for two different experiments, slightly differing in the motion of the elevator motion. Acceleration along the horizontal axis corresponds mostly to noise, while along the vertical axis to the motion of the elevator cabin between floors; first positive acceleration, then motion with nearly constant velocity (no significant acceleration) and finally negative acceleration before stopping. Red and blue indicate the two different smartphone sensors, while a green line the approximate "true" values (known elevator path).

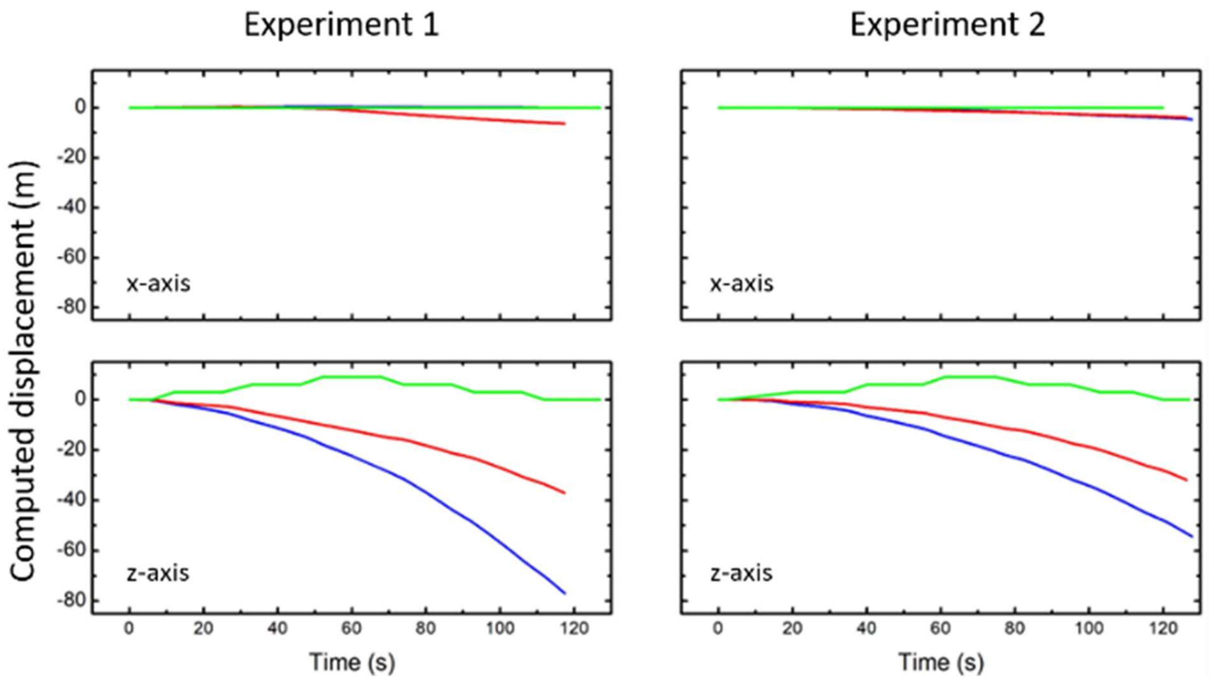

Figure 4. Computed displacements based on the accelerations of Figure 2. Top line indicates computed displacement along a horizontal (x) axis in which no displacement is expected (i.e. computed displacement corresponds to noise). The bottom line shows the movement of the elevator cabin between floors, along the vertical (z) axis. Red and blue indicate the two different smartphone sensors, while a green line the approximate "true" path. Mark the rather random character of drift.

Data Analysis

Data were downloaded from the smartphones in CSV files, which can be readily be analyzed using software such as EXCEL© or ORIGIN©. In the case the resolution of the timing output is only a second, or a tenth of second (i.e. of series of measurements assigned to the same time), timing was derived from interpolation assuming piecewise constant rates. This approximation has not serious impact in the accuracy of results and led to an easily analyzed data set.

Using eq. (2) we computed displacements during several tests involving movement of the elevator cabin from one floor to other. Selected results are plotted in Figures 2 and 3.

Discussion

The results of Fig 4 seem astonishing, because they show that the error ("drift") in the x-axis reaches a few meters, while in the z-axis tens of meters. This is not an unknown effect, and is known as "drift" in earthquake engineering (Fig 1) , and there have been proposed different techniques to remove it, especially from earthquake records: The basic idea is that after an earthquake no permanent motion remains, so that the deflection of the last point is known, and a correction can be made to remove drift. In many cases, the uncorrected displacement is approximated by a first or second order polynomial which is subtracted from the uncorrected displacement, and the residual corresponds to a corrected displacement record (Wang et al, 2003). Another approach is to use signal analysis techniques, for example to remove part of the spectral content of the acceleration record so that the drift in the displacement is removed (Fig 1; Boore and Bommer, 2005).

Figures 3 and 4, however, show that drift has not a deterministic, but a stochastic character: different instruments lead to a different drift (Wang et al, 2003; Moschas et al, 2015; Moschas and Stiros, 2019), and if the experiment is repeated, the displacement history is different; this last observation cannot obviously be made with earthquakes. What is also evident is that error (drift) increases with time (number of observations) in a rather polynomial form, in agreement with eq. (4).

The significance of this equation, derived from the law of error propagation, is that the typical (standard) error of computed displacement can only be within certain limits under certain probability level, and it cannot be more deterministically defined.

Another point is that errors seem much higher along the vertical than the horizontal axis, and this seems contradictory to equation (1), assuming that acceleration errors are equal in all three axes. However, equation (1) is valid under the assumption of random and uncorrelated errors, which is an oversimplification (cf. Moschas and Stiros, 2019). In addition, errors are amplified by high acceleration peaks (Stiros, 2008), as those shown in Fig 3.

A final point is that the observed pattern of errors is not limited/ amplified in the case of miniature MEMS in smartphones, but it is characteristic of all types of accelerometers, as experiments with force-balance (Moschas et al., 2015) and high accuracy MEMS (Moschas and Stiros, 2019) indicate.

Educational Implications

The experiment discussed requires one or two smartphones, a lift, a free-downloadable App and popular software such as EXCEL. Hence in simple form it can be easily repeated by anybody, and it can also be easily modified to be made in a classroom; for example, the smartphones can be tied together in a protective plastic bag and forced to move along a square marked on a floor or a wall, or along vertical room corners, and permit easy and safe experimental education at different levels, and also inspire citizen to contribute in problems of the society (for example concerning earthquakes; Lawrence et al, 2014).

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