# **Coordinates Transformation by "Zoning Method" for Parameters Computing between WGS84 and North Sahara**

# Noureddine KHELOUFI, Algeria

Keywords: Cartography GPS, Land readjustment, positioning

# **SUMMARY**

The global transforms model (bursa -wolf, molodensky-badekas) are the famous method used to cmpute parameter tansformation between geodetic systems. In our case, for a huge territory like algeria, these model are not most appropriate to give a good accuracies over these parameters, because informatuion about local geoid over ellipsoid clarke 1880 is not available. but the two dimension models such as geodetic lines, multiple refression,..., have given best results applied over the algerian area. That's why we are hold to use a new method to compute the tansform parameters with good accuracies, this method is called "zonage" and it concist to delimitate our territory to little zone in order to apply three dimension models (bursa- wolf, molodensky-badekas), in this case we can avoid great geoid undulation wich generate errors on computation of patrameters and give thus a bad accuracy. These parameters while computed, could be used to transform a new point over all the zone, the results are validating by a computation programm called TRANSFOR

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# **1. INTRODUCTION**

The global transforms models (Bursa -Wolf, Molodensky-Badekas) are the famous method used to compute transformation parameters between geodetic systems. In our case, for a huge territory like Algeria, these models are not most appropriate to give a good accuracies over these parameters, because information about local geoid over ellipsoid Clarke 1880 is not available but the two dimension models such as geodetic lines, multiple

That's why we are hold to use a new method to compute the transform parameters with good accuracies, this method is called "zoning" and it consist to delimitate our territory to little zone in order to apply three dimension models (bursa- wolf, molodensky-badekas), in this case we can avoid great geoid undulation which generate errors on computation of parameters and thus give a bad accuracy.

These parameters while computed, we can use them to transform a new point over the entire zone, and the results are validated by a computation program called TRANSFOR.

# 2. GENERAL EXPRESSION OF TRANSFORMATION

regression ,..., have given best results applied over the Algerian area.

 $X_{NS}=T+(1+\Delta k)$ .  $R_{(Ez, Ev, Ex)}$ .  $X_{WGS84}$ 

Où X : position vector. T : translation R rotation around 3 axes  $\Lambda k$ : scale factor variation.

# **3. GLOBAL MODELS**

# 3.1 BURSA WOLF model

It's a (07) seven parameters model with general equation:

$$\vec{X} = \vec{T} + (1 + \Delta k).(I + \Delta R).\vec{x}$$

Where:

X: coordinates vector in the first system (1). x: coordinates vector in the second system (2). I : Identity matrix.

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Matricial expression for one point is :

$$\begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \\ \mathbf{Z} \end{pmatrix} = \begin{pmatrix} \mathbf{T}\mathbf{x} \\ \mathbf{T}\mathbf{y} \\ \mathbf{T}\mathbf{z} \end{pmatrix} + (\mathbf{1} + \Delta \mathbf{k}) \begin{pmatrix} \mathbf{1} & -\mathbf{E}\mathbf{z} & \mathbf{E}\mathbf{y} \\ \mathbf{E}\mathbf{z} & \mathbf{1} & -\mathbf{E}\mathbf{x} \\ -\mathbf{E}\mathbf{y} & \mathbf{E}\mathbf{x} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{pmatrix}$$

By putting the unknown in the same vector, we obtain the BURSA - WOLF model under his matricial form.

 $1^{\circ}$ - For one (01) point :

The BURSA – WOLF model can be writen.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & x & 0 & z & -y \\ 0 & 1 & 0 & y & -z & 0 & x \\ 0 & 0 & 1 & z & y & -x & 0 \end{pmatrix} \begin{pmatrix} Tx \\ Ty \\ Tz \\ \Delta k \\ Ex \\ Ey \\ Ez \end{pmatrix}$$

 $2^{\circ}$  - for n points (n>3): we obtain :

$$\begin{pmatrix} X_{1} - x_{1} \\ X_{2} - x_{2} \\ \dots \\ X_{n} - x_{n} \\ Y_{1} - y_{1} \\ Y_{2} - y_{2} \\ \dots \\ Y_{n} - y_{n} \\ Z_{1} - z_{1} \\ Z_{2} - z_{2} \\ \dots \\ Z_{n} - z_{n} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & x_{1} & 0 & z_{1} - y_{1} \\ 1 & 0 & 0 & x_{2} & 0 & z_{2} - y_{2} \\ \dots \\ 1 & 0 & 0 & x_{n} & 0 & z_{n} - y_{n} \\ 0 & 1 & 0 & y_{1} - z_{1} & 0 & x_{1} \\ 0 & 1 & 0 & y_{2} - z_{2} & 0 & x_{2} \\ \dots \\ 0 & 1 & 0 & y_{n} - z_{n} & 0 & x_{n} \\ 0 & 0 & 1 & z_{1} & y_{1} - x_{1} & 0 \\ 0 & 0 & 1 & z_{2} & y_{2} - x_{2} & 0 \\ \dots \\ 0 & 0 & 1 & z_{n} & y_{n} - x_{n} & 0 \end{pmatrix} \begin{pmatrix} Tx \\ Ty \\ Tz \\ \Delta k \\ Ex \\ Ey \\ Ez \end{pmatrix}$$

This system can be resolved by the least squares method.

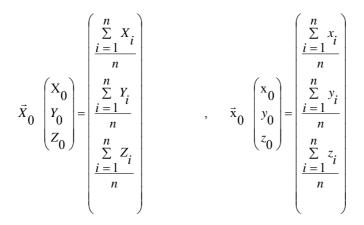
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### **3.2 MOLODENSKY-BADEKAS Model**

The MOLODENSKY-BADEKAS model in Cartesian formalism is obtained by insertion of vector defining an initial point around wich the three rotations (Ex, Ey, Ez) are carried out. This point is considered as a gravity center usually called « isobarycenter », in the both coordinates systems.

This model is generally used for local transformations like cadastral ones.

We take as an initial point, the gravity center expressed in the two (02) systems:



System n°1

Systm n°2

With difference of BURSA-WOLF, MOLODENSKY-BADEKAS introduce in his model, an initial point such as:

$$\vec{X} = \vec{X}_0 + \vec{T} + (1 + \Delta k).(I + \Delta R).(\vec{x} - \vec{x}_0)$$

It's a seven (07) parameters model, with general equation:

where : X: coordinates vector in the first system (1). x: coordinates vector in the second system (2). X<sub>0</sub>: coordinates vector of gravity center. I : Identity matrix.

The matricial writing for one point is :

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} Tx \\ Ty \\ Tz \end{pmatrix} + (1 + \Delta k) \begin{pmatrix} 1 & -Ez & Ey \\ Ez & 1 & -Ex \\ -Ey & Ex & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

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With putting  $(\vec{x} - \vec{x}_0) = \vec{x}$  and with ordonnate the matrix, MOLODENSKY-BADEKAS model become for n points:

$ \begin{pmatrix} X_1 - X_1 \\ X_2 - X_2 \end{pmatrix} $	$ \begin{pmatrix} 1 & 0 & 0 & \overline{x}_1 & 0 & \overline{z}_1 - \overline{y}_1 \\ 1 & 0 & 0 & \overline{x}_2 & 0 & \overline{z}_2 - \overline{y}_2 \end{pmatrix} $	
$\begin{bmatrix} \dots & \dots & \dots \\ \mathbf{X}_{n} - \mathbf{X}_{n} \end{bmatrix}$	$\begin{bmatrix} \cdots & \cdots & \cdots \\ 1 & 0 & 0 & \overline{x}_n & 0 & \overline{z}_n & -\overline{y}_n \end{bmatrix}$	(Tx)
$\begin{vmatrix} Y_1 - y_1 \\ Y_2 - y_2 \end{vmatrix} =$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Ty Tz Δk
$\mathbf{Y}_{n} - \mathbf{y}_{n}$	$\begin{bmatrix} \cdots & \cdots & \cdots \\ 0 & 1 & 0 & \overline{y}_n - \overline{z}_n & 0 & \overline{x}_n \end{bmatrix}$	Ex Ey
$ \begin{array}{c} Z_1 - Z_1 \\ Z_2 - Z_2 \end{array} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	(Ez)
$\left( \begin{array}{c} \dots \\ \mathbf{Z}_{n} - \mathbf{Z}_{n} \end{array} \right)$	$ \begin{pmatrix} \cdots & \cdots & \cdots \\ 0 & 0 & 1 & \overline{z}_n & \overline{y}_n & -\overline{x}_n & 0 \end{pmatrix} $	

## 2. «ZONING» METHOD PRESENTATION

### 2.1 The method of treatment with "Zoning"

For the three dimensions models, only the applications on large scales gave good results, one can explain that by the fact thus that the propagation of errors on the large geodetic ones affect the accuracy determination of the co-ordinates of the points after Transformation, one could obtain relatively large residues for the application on a small scale (case of Algeria). On the other hand, for the cadastral applications (great scales), all the models were considered to be good bus they gave geodetic accuracies details (centimetric).

Thus for the cadastral applications, all the models give satisfactory results.

The introduction of the new transformation (approach of the geodetic lines) made it possible on the other hand to obtain guarantors results with the geodetic requirements (15 à 20 cm in average) some is the extent (case of Algeria) with the proviso of taking account of the correction of azimuth and scale.

It is noticed that for the applications on large scales, the similarity planes was sufficient for this kind of application (the networks GPS and the Northern Sahara are almost in conformity).

Any time it is to be announced that this work approaches the two-dimensional aspect by highlighting the majority of the methods of transformations such as the multiple regression, the transformation closely connected and the geodetic lines. The third dimension which is a fundamental unknown factor, is approached in the two three-dimensional models (Bursa Wolf

zoning, Molodensky-Badekas zoning) but being identified with the ellipsoidic height, because of the lack of information on the height of the geoïd relating to the ellipsoid Clarke 1880, and following the coherence which exists between the co-ordinates of the points, the accuracies details on the results resulting from these two models are faded.

It is thus necessary to carry out a zoning of the territory in question to avoid especially the problem of the great undulations geoid for the three dimensions models.

The traditional methods, currently replaced new methods faster and because of their completion period and of the cost of work. The integration of the GPS in geodesy, registers, topography... will make it possible to obtain a saving of time and precision and will reduce considerably work of ground (recognition, duration of observations...).

# 3. DESCRIPTION OF THE PROGRAM "TRANSFOR" WITH ZONING

D years the principal window of program TRANSFOR, a third button was added, called "3D zonage transform", it gives the possibility of selecting total models with zoning while returning us to the button "models 3d-zoning" which makes it possible in its turn to make the choice between the two total models "Bursa Wolf", "Molodensky Badekas", whose source codes were implemented by routines of zoning, finally, the model being selected, one has the choice between several rays "going from 25 to 200 km and a free choice of another value of the ray by using on the button "other values", which once clicked, a window of dialogue appears Oû one can enter a value to the choice of R, all these radio operator buttons are placed on a panel named "rayons zoning", this part added with the program makes it possible to delimit a territory extended such as Algeria in small zones following of the selected rays and to make the transformations by applying the three-dimensional models (Bursa Wolf, Molodensky, etc.). Results were obtained on each zone estimating the residues and the precise details for each model and on delimited zones.

# 3.1 Main Window of the Program

This window called ray of zoning makes it possible during the execution to make the choice and to return a value to the choice of the ray for which one carries out calculation. These values vary between 50 km and 200 km in our case with an option to enter an unspecified value for which the program makes the choice of the points of support which are in a ray of a value equal or lower than the selected ray. The point thus chosen, the program takes into account its co-ordinates geographical (or Cartesian, that depends on the choice) in calculation, thus one makes the treatment on a reduced zone (cadastral), and one thus avoids the great undulations geoid which especially affect the systematism of treatment for the total models. A page of treatment is published O  $\hat{u}$  one posts the files of selected points of support, the parameters of transformation calculated for zoning as well as a statistical analysis giving the values of the "Rms" on the "Easting" and "Northing" for the points of support and the points of control, as well as the new points transformed by zoning.

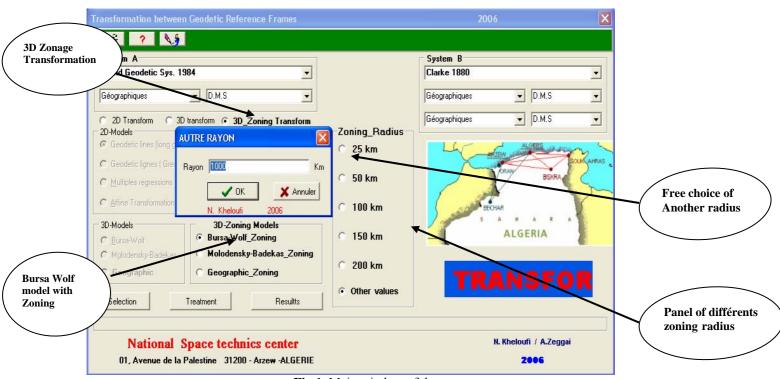


Fig.1 Main window of the progam.

## 3.2 Edition of treatment Results

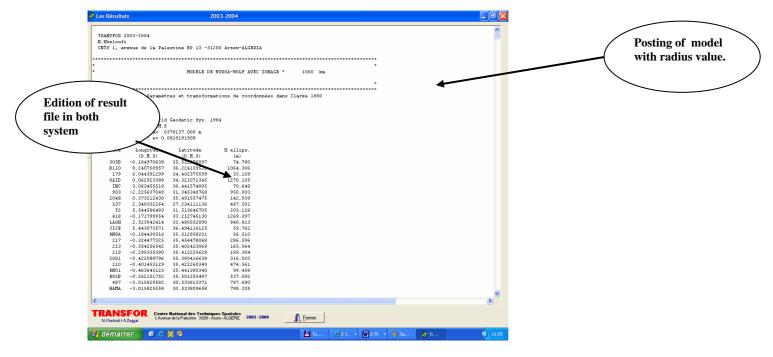


Fig 2. Edition of treatment results files.

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# **Program Structure.**

TRANSFOR manages the modules implemented in the following architectural form:

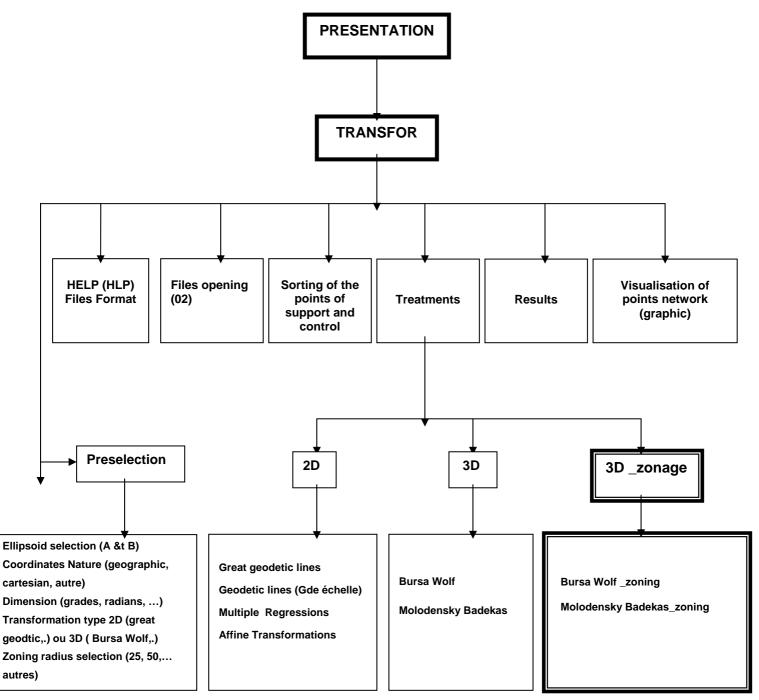


Figure 3. Structure of Program with implementation of zonage

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## 4- RESULTS AND ANALYSIS.

#### 4.1 Global Models

#### 4.1.1 Bursa Wolf

The table hereafter summarizes the statistical results of the treatment by the model of Bursa – Wolf with zoning, one defers here the RMS relating to the various radius of zoning used in the program (25 to 200 km).

MODELS	RADIUS (KM)	COMPONENTS	MINIMUM (M)	MAXIMUM (M)	AVERAGE (M)	RMS (M)
	25	X Y Z	-0.044 -0.098 -0.064	0.006 0.089 0.055	0.0085 0.0000 0.0000	0.028 0.070 0.042
	50	X Y Z	-0.548 -0.114 -0.321	0.483 0.061 0.278	0.0000 0.0000 0.0611	0.367 0.058 0.236
	100	X Y Z	-0.548 -0.114 -0.321	0.483 0.061 0.278	0.0000 0.0000 0.0611	0.367 0.058 0.236
Bursa Wolf	150	X Y Z	-0.156 -0.119 -0.186	0.210 0.047 0.226	0.0000 0.0000 0.0000	0.121 0.064 0.133
	200	X Y Z	-0.782 -0.178 -0.523	0.436 0.167 0.258	-0.000125 0.000125 0.015750	0.389 0.107 0.246
	Autre (250)	X Y Z	-0.693 -0.156 -0.464	0.455 0.194 0.276	0.00000 0.03600 -0.000142	0.379 0.100 0.238

**Table 1-1 :** BURSA-WOLF Model Results Analysis : support point Residues.

### 4.1.2 <u>Molodensky-Badekas</u>

Ce tableau illustre pour le modèle de Molodensky les résultats statistiques issues du traitement par le programme.

MODELS	RADIUS (KM)	COMPONENTS	MINIMUM (M)	MAXIMUM (M)	AVERAGE (M)	RMS (M)
	25	X Y Z	-0.044 -0.098 -0.064	0.006 0.089 0.055	0.0085 0.0000 0.0000	0.028 0.070 0.042
	50	X Y Z	-0.548 -0.114 -0.321	0.483 0.061 0.278	0.0000 0.0000 0.0611	0.367 0.058 0.236
	100	X Y Z	-0.548 -0.114 -0.321	0.483 0.061 0.278	0.0000 0.0000 0.0611	0.367 0.058 0.236
MOLODENSK Y	150	X Y Z	-0.156 -0.119 -0.186	0.210 0.047 0.226	0.0000 0.0000 0.0000	0.121 0.064 0.133
	200	X Y Z	-0.782 -0.178 -0.523	0.436 0.167 0.258	-0.000125 0.000125 0.015750	0.389 0.107 0.246
	Autre (250)	X Y Z	-0.693 -0.156 -0.464	0.455 0.194 0.276		0.379 0.100 0.238

Chart 2.1: Bursa Wolf model with out zoning.

### 4.2 Comparative Analysis between the Various Rays of Zoning.

To set the ideas on the evolution of the values of the RMS according to the rays of zoning, one takes with the choice a minimal value and a maximum value with another intermediate value and one traces for each radius, a statistical table including/understanding the minimum, the maximum, average and RMS.

Model	Components	Minimum	Maximum	Average	RMS
	Х	-3.228	2.701	-0.0001	1.989
Bursa Wolf	Y	-0.990	1.193	0.0001	0.679
	Z	-1.663	1.045	-0.0002	1.172

#### 4.2.1 Bursa- Wolf with out zoning applied over Algerian territory

Table 1.2 : Bursa Wolf Model with out zoning.

The model of Bursa Wolf without zoning is treated in memory of master (2004), the table of top shows how the RMS are of order metric for three components X, Y and Z, because applied to a territory extended such as Algeria with deteriorations of the network as well as a lack of information on the geoid, these precise details are not geodetic.

# 4.2.2 Bursa- Wolf with zoning (case of Algeria)

	1-	Radius	equal	25	Km.
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Model	Components	Minimum	Maximum	Average	RMS
	Х	-0.044	0.006	0.0085	0.028
Bursa Wolf	Y	-0.098	0.089	0.0000	0.070
	Z	-0.064	0.055	0.0000	0.042

Table 1.3: Bursa Wolf Model with zoning (25 km).

Table 2-2, shows well that the use of zoning (Ray equalizes 25 km) gives a better result; the precise details are centimetric, therefore geodetic. This returns to the fact that the application of a three model in fact Bursa-Wolf on a reduced zone, compensates for the lack of information on the geoid and thus gives RMS acceptable.

2- Radius equa	l 50 Km.
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Model	Components	Minimum	Maximum	Average	RMS
	Х	-0.548	0.483	0.0000	0.367
Bursa Wolf	Y	-0.114	0.061	0.0000	0.058
	Z	-0.321	0.278	0.0611	0.236

Table 1.4: Bursa Wolf model with zoning (50 Km).

It is noticed that for a radius zoning of 50 km, the precise details remain always centimetric but of a higher order than that of twenty five kilometers.

#### 3- Radius equal 250 Km.

Model	Components	Minimum	Maximum	Average	RMS
	Х	-0.693	0.455	0.00000	0.379
Bursa Wolf	Y	-0.156	0.194	0.03600	0.100
	Z	-0.464	0.276	-0.000142	0.238

**Table 1.5:** Bursa Wolf with zoning (250 Km).

One increases the radius of zoning up to 250 K m, we obtain precise details geodetic, higher has those of the lower radius (25 and 50km), which let us conclude that with the fu R and as the ray of zoning increases, L are precise details increases NT until reaching a metric order, in this case one is in extreme cases of a model without Zoning.

The table which follows illustrates these remarks.

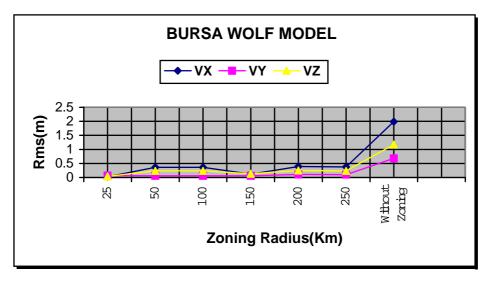


Fig.4: residues of the radius of zoning

### 4.3 Molodensky Badekas with out zoning (case of Algeria)

Model	Components	Minimum	Maximum	Average	RMS
	Х	-3.228	2.701	-0.0001	1.989
Bursa Wolf	Y	-0.990	1.193	0.0001	0.679
	Z	-1.663	1.045	-0.0002	1.172

 Table 1.2 : Molodensky model with out zoning.

The model of Bursa Wolf without zoning is treated in memory of master (2004), the table of top shows how the RMS are of order metric for three components X, Y and Z, because

applied to a territory extended such as Algeria with deteriorations of the network as well as a lack of information on the geoid, these precise details are not geodetic.

## 4.3.1. Molodensky Badekas avec zonage appliqué sur l'Algérie

#### 1- Rayon de 25 Km.

Model	Components	Minimum	Maximum	AVERAGE	RMS
	х	-0.044	0.006	0.0085	0.028
Bursa Wolf	Y	-0.098	0.089	0.0000	0.070
	Z	-0.064	0.055	0.0000	0.042

Tableau 1.3 : Molodensky model with zoning (25 Km).

Table 2-2, shows well that the use of zoning (Ray equalizes 25 km) gives a better result, the precise details are centimetric, therefore geodetic. This returns to the fact that the application of a three dimensions model in fact Bursa-Wolf on a reduced zone compensates for the lack of information on the geoid and thus gives RMS (Quadratic Average Errors) acceptable.

2- Rayon de 50 Km.
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Model	Components	Minimum	Maximum	Average	RMS
Bursa Wolf	х	-0.548	0.483	0.0000	0.367
	Y	-0.114	0.061	0.0000	0.058
	Z	-0.321	0.278	0.0611	0.236

Tableau 1.4 : Modèle de Molodensky avec zonage Rayon de 50 Km.

It is noticed that for a ray of zoning of 50 km, the precise details remain always centimetric but of a higher order than that of twenty five kilometers.

#### 3- Radius equal 250 Km.

Model	Components	Minimum	Maximum	Average	RMS
Bursa Wolf	X	-0.693	0.455	0.00000	0.379
	Y	-0.156	0.194	0.03600	0.100
	Z	-0.464	0.276	-0.000142	0.238

Tableau 1.5: Modèle de Molodensky avec zonage Rayon de 250 Km.

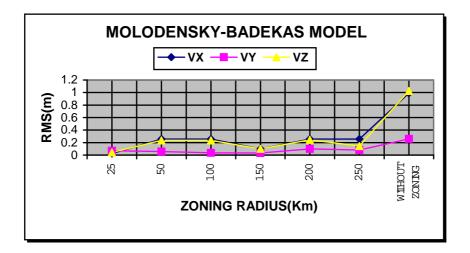


Fig 4 : Residues of zoning radius.

We increase the ray of zoning up to 250 K m, one obtains precise details geodetic, higher has those of the lower rays (25 and 50km), which let us conclude that as the ray of zoning increases the order of the precise details increases until reaching a metric order, in this case one is in extreme cases of a model without Zoning. The table which follows illustrates these remarks.

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### **CONTACTS**

Attached of Research Noureddine Kheloufi National Space Technics Center BP 13, Rue de Palestine Arzew **ALGERIA** Tel. + 213 41 47 22 17 Fax + 213 41 47 36 65 OR 48 59 49 44 Email: nouri.kh@caramail.com Web site:www.cnts.dz