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Towards an Advanced Estimation of Measurement Uncertainty Using Monte-Carlo Methods- Case Study Kinematic TLS Observation Process

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Motivation

- Main engineer tasks are design, produce, and test of structures and processes
- Mathematical and physical modelling of the different phenomena are needed
- Some information about constants, parameters, and functional variables are necessary

➔ Sources of uncertainties :

- From the measurements,
- From statistical evaluations of the model and
- From the model itself

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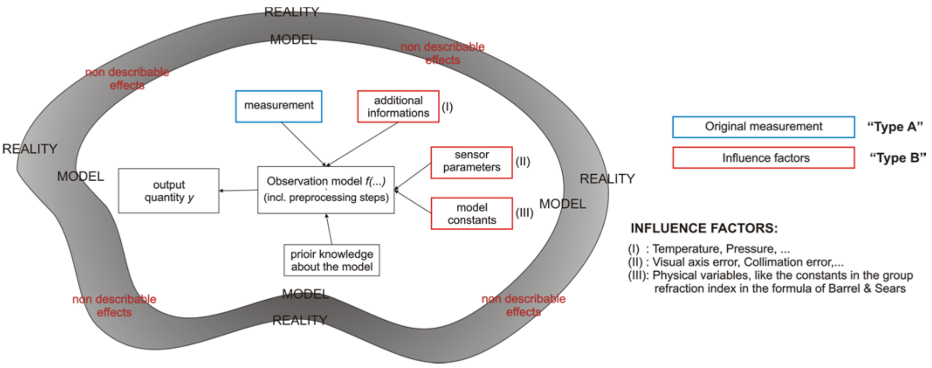
Motivation

Influence variables for the uncertainties of measurements



Different types/sources of uncertainty have to be taken into account in geodetic data analysis.

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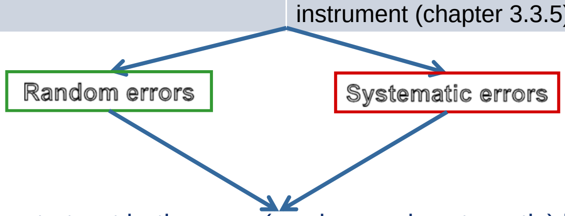
GUM strategies

GUM

The Guide to the expression of Uncertainty in Measurement groups the occurring uncertainties into „Type A" and „Type B".

"Type A" uncertainties…	"Type B" uncertainties…
…are determined with the classical statistical methods (chapter 3.3.5)	…are subject to other uncertainties like experience with and knowledge about an instrument (chapter 3.3.5)



GUM proposes to treat both errors (random and systematic) in a stochastic framework and introduces variances to describe their uncertainties

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Uncertainty Modelling

Mathematical backgrounds for uncertainty

- Approximation theory
- Stochastics
- Bayes theory: Monte Carlo
- Dempster-Shafer theory
- Interval mathematics
- Fuzzy theory

Tasks
- Estimation
- Filtering
- Prediction
- Testing

Modeling, treatment, propagation of uncertainty

Tools?

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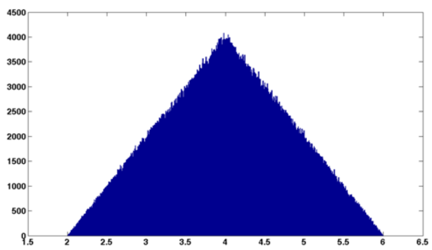
Uncertainty Modelling-Bayesian strategy (MC)

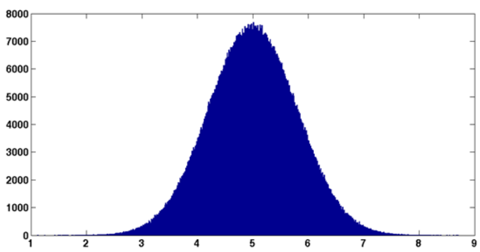
Uncertainty Modelling with Monte Carlo techniques:

$$y = f(z_1, z_2, \ldots, z_n) = f(\mathbf{z})$$

Step 1: Generate M random samples (i=1…M) of $z_j^{(i)}$ from its probability density function (pdf)



Triangular distribution (M=1,000,000)



Normal distribution (M=1,000,000)

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Uncertainty Modelling-Bayesian strategy (MC)

Uncertainty Modelling with Monte Carlo techniques:

$$y = f(z_1, z_2, \ldots, z_n) = f(\mathbf{z})$$

Step 1: Generate M random samples (i=1…M) of $z_j^{(i)}$ from its pdf

Step 2: Compute the output quantity (estimate of the pdf for y)

$$y^{(i)} = f(z_1^{(i)}, z_2^{(i)}, \ldots, z_n^{(i)}) = f(\mathbf{z}^{(i)})$$

Step 3: Compute the estimated expectation of the output quantity

$$\hat{E}(y) = \hat{E}(f(\mathbf{z})) = \frac{1}{M}\sum_{i=1}^{M} f(\mathbf{z}^{(i)})$$

Step 4: Compute the estimated variance of the output quantity

$$V(\hat{E}(y)) = \hat{\sigma}_y^2 = \frac{1}{M}\sum_{i=1}^{M} (f(\mathbf{z}^{(i)}) - \hat{E}(f(\mathbf{z})))(f(\mathbf{z}^{(i)}) - \hat{E}(f(\mathbf{z})))^T$$

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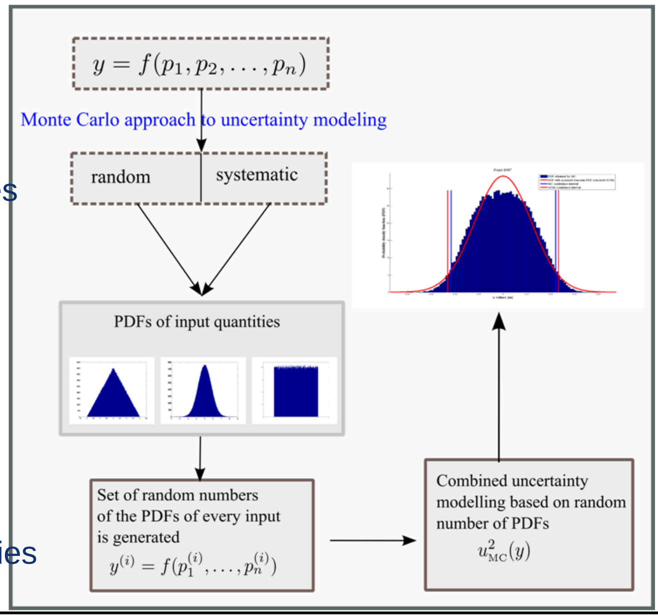
Computation steps: Bayesian Strategy.MC simulation

1 Modelling of uncertainty for input quantities

2 Choice of pdf

3 Evaluating uncertainty for output quantities

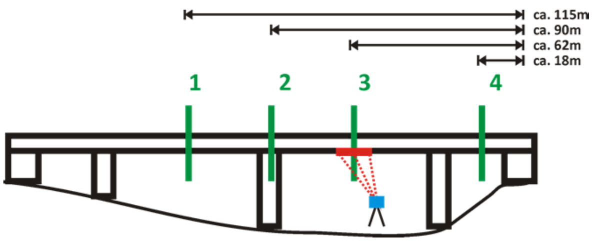


Object and setup

Autobahn bridge in southern Germany

Experiment: Deformations due to defined traffic loads

- Static loads in four positions, dynamic loads
- Monitoring in all spatial modes: 3D / 2D / 1D
- Fixed scanner position



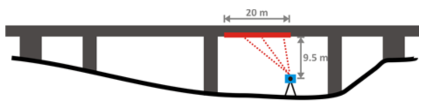
Scanner in use: Z+F Imager 5006

Monte-Carlo simulation and validation

Simulation of 2D k-TLS profile observations

Purpose

- Planning and modeling
- Analysis and diagnosis



Unloaded state:
500 profiles
12.5 profiles/s
7216 pts/profile

Approach: Reproduction of the real situation by

- identical geometrical configuration
- identical repetition rate
- randomly varying observation values (Monte-Carlo)
  - Distance: constant metric component
  - Distance: distance proportional metric component
  - Zenith angle: constant angular component

500 samples per random quantity

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Monte-Carlo simulation and validation

Functional model of the k-TLS profiles

$$z = d \times \cos(z)$$

Simulation parameters

Simulation I:		
Input quantity	Prob. density	Num. value (std.dev.)
Distance: constant	Normal	0.5 mm
Distance: proportional	Normal	30 ppm
Zenith angle	Normal	10 mgon

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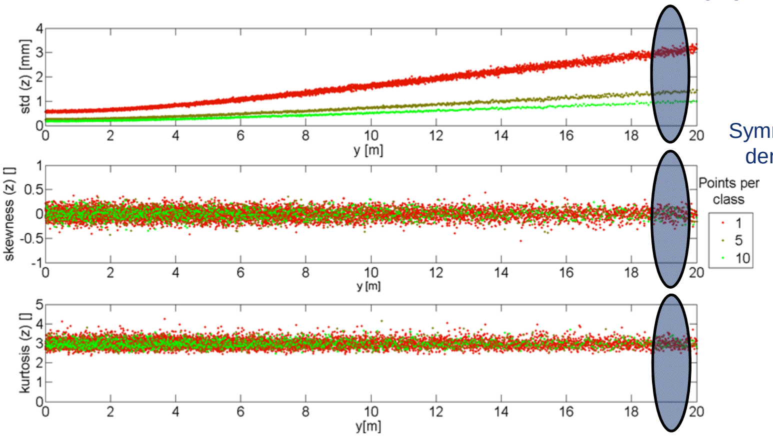
Monte-Carlo simulation and validation

Results of Simulation I – statistical moments

Square-root law of averaging



Symmetric density

Kurtosis of Normal distribution

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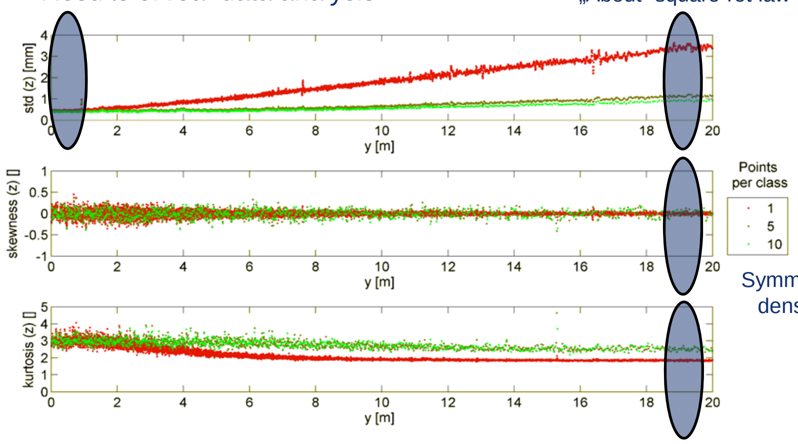
Monte-Carlo simulation and validation

Results of real-data analysis

„About“ square-rot law



Symmetric density

Kurtosis smaller than Normal distribution!

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Monte-Carlo simulation and validation

Functional model of the k-TLS profiles

$$z = d \times \cos(z), \; z = z_0 + Dz$$

Simulation parameters

Simulation I:		
Input quantity	Prob. density	Num. value (std.dev.)
Distance: constant	Normal	0.5 mm
Distance: proportional	Normal	30 ppm
Zenith angle	Normal	10 mgon

Simulation II: Including vertical step motor		
Input quantity	Prob. density	Num. value (std.dev.)
Distance: constant	Normal	0.3 mm
Distance: proportional	Normal	30 ppm
Zenith angle	Normal	5 mgon
Vertical increment	**Uniform**	20 mgon

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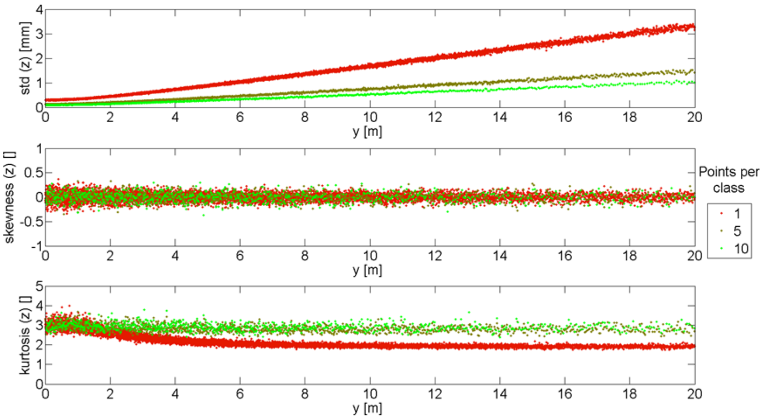
Monte-Carlo simulation and validation

Results of Simulation II – statistical moments



Comprehensive reproduction of the real-data results !

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Conclusions

- Simulation of observation processes is important for both pre-analysis and post-analysis.
  - Monte-Carlo techniques are effective and easy-to-implement.
  - An extended error model is required for a meaningful simulation.
- Real-data series give evidence for non-normal random influences.
  - Filtering techniques mitigate such deviations.
  - More refined models and statistical analyses are required.

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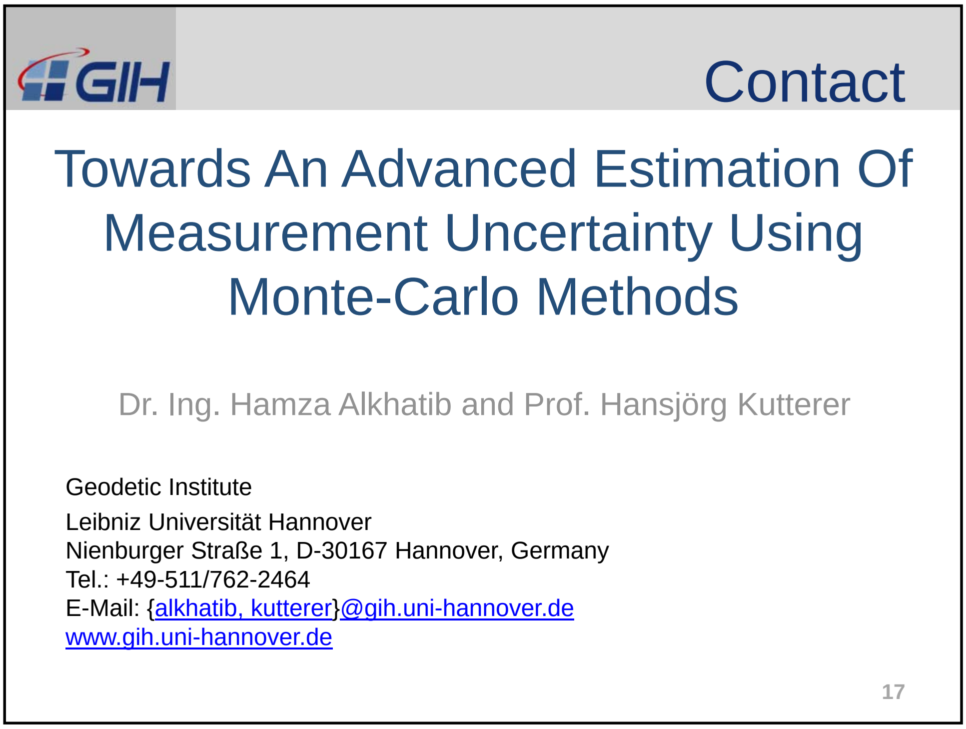
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